# A Dual-Band Unequal Wilkinson Power Divider With Arbitrary Frequency Ratios

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***Abstract*—This letter presents a theoretically rigorous investigation and experimental verification for an asymmetrical, unequal power divider working at dual frequency bands. Through the even- and odd-mode analysis that can be applied to dual-band, unequal power divider by assuming virtual open and short points, analytically exact solutions have been derived for the design parameters values. For verification of this analysis, commercially available software including ADS and CST MWS based on circuit theory and full-EM theory, respectively, and already-established equations have been adopted.**

***Index Terms*—Dual-band, even-odd mode, unequal power divider, Wilkinson power divider.**

## I. Introduction

As a predominant element used for combining and dividing the power in the microwave system, the Wilkinson power divider has been proposed by E. Wilkinson in 1960 [1]. This device has some advantages such as reciprocal characteristics and excellent isolation level between two output ports. In the usual circuit applications, it is operated at a single frequency band with quarter wavelength transmission lines.

In addition to the research on equal Wilkinson power divider, the dual-frequency Wilkinson power divider operating at two arbitrary frequencies (GSM & PCS) has been proposed recently [2]. Furthermore, a tri-band Wilkinson power divider using multi-section transmission lines has been reported [3]. The previously proposed power dividers have a symmetrical structure and are easily analyzed by employing the conventional even- and odd-mode analysis for equal power division [4].

According to the general purpose of power divider/combiner, new types of unequal power divider have been studied by many researchers. Recently, an unequal power divider with dual-frequency characteristic has been proposed [5], [6]. However, the proposed analysis in [5] has a disadvantage of fixed frequency ratio such as 1:2. In addition, the complexity in exact analysis increases when two frequencies are employed. Even- and odd-mode analysis adopted in [6] has reduced the complexity in analysis, but leads to an imperfect isolation by employing a direct connection of $Z_0$ at output ports. In this letter, the usefulness of even- and odd-mode analysis for an asymmetrical structure with variable power splitting-ratios at dual bands are treated by suggesting the exact formulas based on even- and odd-mode analysis and providing the design parameters values for a dual-band, unequal Wilkinson power divider with arbitrary frequency ratios.

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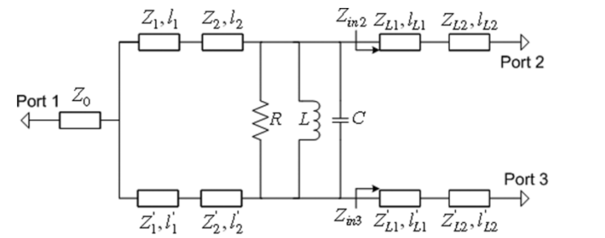


Fig. 1. Unequal Wilkinson power divider with dual-band and arbitrary frequency ratios.

## II. Even-Odd Mode Analysis

In the case for an unequal power divider with asymmetrical structures as shown in Fig. 1, transmission or admittance matrix approach has been used for circuit analysis [5], [7], [8]. However, in the analysis using transmission matrix or admittance matrix, the complexities in the parameter evaluation increase with repeated simultaneous equations resulting in more complexities in the case of multi-section and multi-band Wilkinson power divider.

Hence, it is proposed that the concept of even- and odd-mode analysis is not limited to the symmetrical structure and extended to the asymmetrical structure with the simplified evaluation process.

### A. Even-Mode Analysis

The key point in the even-mode analysis is to enforce appropriate sources that result in no current flow through the passive components connected between ports 2 and 3 so that the overall structure in even-mode can be divided into simple two circuits at each port as shown in Fig. 2. In order to obtain the required power ratios ($K^2 = P_3/P_2$) at ports 2 and 3, the input impedances looking into the ports 2 and 3 at marker "A" and "B" should satisfy (1) because the voltages at marker "A" and "B" are equal

$$Z_a = K^2 Z_b, \quad Z_{in2} = K^2 Z_{in3}. \tag{1}$$

At this time, the condition that $Z_a$ and $Z_b$ in parallel should be matched to the characteristic impedance $Z_0$ leads to the derivation of the exact values of $Z_a$ and $Z_b$ rewritten as

$$Z_a = (1+K^2)Z_0, \quad Z_b = \frac{1+K^2}{K^2}Z_0. \tag{2}$$

By selecting $Z_{in2} = KZ_0$ and $Z_{in3} = Z_0/K$ and assuming the circuit in Fig. 2 as two section transformer with input

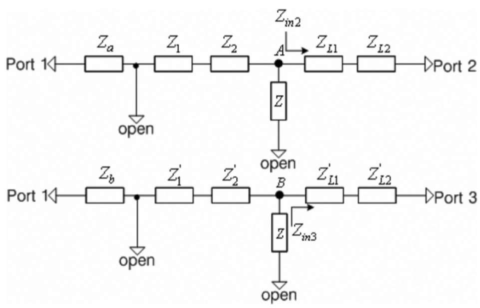


Fig. 2. Equivalent circuits of even-mode analysis.

impedance $Z_a$ and load impedance $Z_{in2}$, the corresponding lengths and the characteristic impedances of transmission lines must be calculated as

$$l_1 = l_2 = \frac{n\pi}{\beta_1+\beta_2} = l_1' = l_2' \quad (3)$$

$$Z_2 = Z_0\sqrt{\frac{K}{2\alpha}(K^2-K+1)+\sqrt{\left[\frac{K}{2\alpha}(K^2-K+1)\right]^2+K^3(1+K^2)}} \quad (4)$$

$$Z_1 = \frac{K(1+K^2)Z_0^2}{Z_2} \quad (5)$$

at the selected two frequencies based on the theory introduced in [9] where $\alpha = (\tan\beta_1 l)^2$.

The right-handed side of marker "A" can also be considered as a two-section transformer with input impedance $Z_{in2}$ and load impedance $Z_0$, resulting in

$$l_{L1} = l_{L2} = \frac{n\pi}{\beta_1+\beta_2} = l_{L1}' = l_{L2}', \quad (6)$$

$$Z_{L2} = Z_0\sqrt{\frac{1}{2\alpha}(K-1)+\sqrt{\left[\frac{1}{2\alpha}(K-1)\right]^2+K}},\ Z_{L1} = \frac{KZ_0^2}{Z_{L2}}. \quad (7)$$

In a similar way, the corresponding lengths and characteristic impedances at port 3 can be evaluated.

If we assume the selected two frequencies as $f_2 = 2f_1$, each length of microstrip line in (3) and (6) must be $\lambda/6$. In addition, it is ensured that all the characteristic impedances derived in even-mode analysis converge to (5)–(8) listed in [5].

## B. Odd-Mode Analysis

By inserting the input voltages at ports 2 and 3 to make the marker "A" and "B" have opposite polarity, the overall structure in odd-mode can also be divided into simple circuits as shown in Fig. 3 with virtual ground at a certain point. In Fig. 3, $x$, $a$, $b$, $x'$, $a'$, and $b'$ are the unknowns for the circuit division of passive components supporting the enhancement of isolation performance between two output ports. The unknowns must simultaneously satisfy the following relationships between the original circuit components

$$xR + x'R = R,\ bL + b'L = L,\ \frac{a}{C} + \frac{a'}{C} = \frac{1}{C}. \quad (8)$$

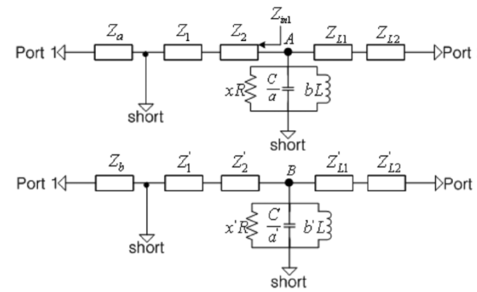


Fig. 3. Equivalent circuits of odd-mode analysis.

TABLE I
COMPARISONS OF CIRCUIT PARAMETERS VALUES ACCORDING TO THE DIFFERENT POWER SPLITTING RATIOS (A) BETWEEN THIS WORK AND [5], (B) BETWEEN THIS WORK AND [2]: $f_2 = 2f_1$

| $K^2$=2 | (17) in this work | | (19) and (20) in [5] | |
|---|---|---|---|---|
| $f_1$ [GHz] | L [nH] | C [pF] | L [nH] | C [pF] |
| 0.5 | 30.17 | 1.68 | 30.17 | 1.68 |
| 1 | 15.09 | 0.84 | 15.09 | 0.84 |
| 1.5 | 10.06 | 0.56 | 10.06 | 0.56 |
| 2 | 7.54 | 0.42 | 7.54 | 0.42 |
| 2.5 | 6.03 | 0.34 | 6.03 | 0.34 |

(a)

| $K^2$=1 | (17) in this work | | (20) and (21) in [2] | |
|---|---|---|---|---|
| $f_1$ [GHz] | L [nH] | C [pF] | L [nH] | C [pF] |
| 0.5 | 28.26 | 1.79 | 28.17 | 1.78 |
| 1 | 14.13 | 0.90 | 14.09 | 0.89 |
| 1.5 | 9.42 | 0.60 | 9.39 | 0.59 |
| 2 | 7.06 | 0.45 | 7.04 | 0.45 |
| 2.5 | 5.65 | 0.36 | 5.63 | 0.36 |

(b)

As expected from Fig. 3, since the impedance $Z_a$ has no effect on the circuit analysis due to the virtual ground at a certain point, the impedance seen looking into the port 1 from the marker "A" is evaluated as

$$Z_{in1} = jZ_2\frac{(Z_1+Z_2)\tan(\beta l)}{Z_2-\alpha Z_1}. \quad (9)$$

From the matching condition enforced to marker "A" connected with $Z_{in1}$ and the impedance looking into the passive components, we can obtain the following simple equation:

$$\left(\frac{1}{Z_{in1}}+\frac{1}{xR}+j\left(\omega\frac{C}{a}-\frac{1}{\omega bL}\right)\right)^{-1} = KZ_0 = Z_{in2} \quad (10)$$

which can be separated by

$$R = \frac{KZ_0}{x}, \quad (11)$$

$$\frac{Z_2-\alpha Z_1}{Z_2(Z_1+Z_2)\tan\beta l} = \omega\frac{C}{a} - \frac{1}{\omega bL}. \quad (12)$$

Since (12) must be working for two target frequencies, the passive components, $L$ and $C$ can be obtained from the simultaneous equations with (14)

$$L = \frac{1}{bP(\omega_2+\omega_1)}\left(\frac{\omega_1}{\omega_2}-\frac{\omega_2}{\omega_1}\right),\ C = aP\left(\frac{1}{\omega_2}+\frac{1}{\omega_1}\right)\frac{\omega_1\omega_2}{\omega_1^2-\omega_2^2} \quad (13)$$

$$P = \frac{Z_2-\alpha Z_1}{Z_2(Z_1+Z_2)\tan\beta_1 l}. \quad (14)$$

Hence, using the similar expression for port 3 and (8), it is expected that the finally derived passive components can be

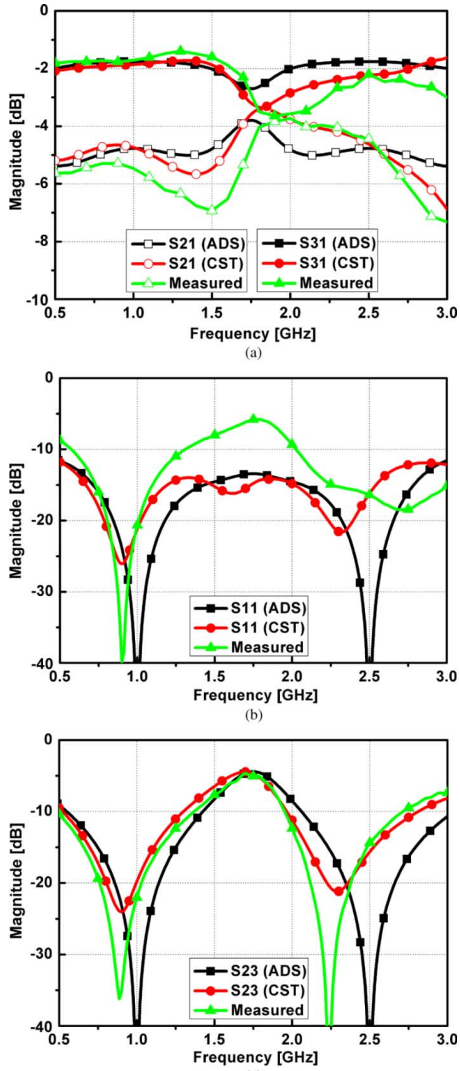


Fig. 4. Simulation and measured results when $f_1 = 1$ GHz, $f_2 = 2.5$ GHz, and $K^2 = 2$: (a) S21 and S31, (b) S11, and (c) S23.

represented as functions of frequency, power ratios, the corresponding length and characteristic impedances previously obtained from the even-mode analysis

$$R = \frac{1+K^2}{K} Z_0, \tag{15}$$

$$L = \frac{1+K^2}{PK^2} \frac{1}{\omega_2+\omega_1} \left(\frac{\omega_1}{\omega_2} - \frac{\omega_2}{\omega_1}\right), \tag{16}$$

$$C = \frac{PK^2}{1+K^2} \frac{\omega_1\omega_2}{\omega_1^2-\omega_2^2} \left(\frac{1}{\omega_2} + \frac{1}{\omega_1}\right). \tag{17}$$

Table I describes the comparison of our analysis with other published method. Table I shows the accuracy and convergence of our data compared with results obtained from transmission matrix in unequal division of [5] and conventional odd-mode analysis in equal division, respectively, by assuming the two target frequencies as $f_2 = 2f_1$ and different power splitting ratios ($K^2 = 2$ and $K^2 = 1$).

## III. Simulations and Measured Results

The calculated parameters values obtained from (3)–(7) and (15)–(17) satisfying the requirements of different frequency ratio ($f_1 = 1$ GHz, $f_2 = 2.5$ GHz) and different power division ratio ($K^2 = 2$) are summarized as

$$Z_1 = 110\,\Omega,\ Z_2 = 96\,\Omega,\ Z_1' = 55\,\Omega,\ Z_2' = 48\,\Omega, \tag{18}$$

$$Z_{L1} = 61\Omega,\ Z_{L2} = 58\,\Omega,\ Z_{L1}' = 41\,\Omega,\ Z_{L2}' = 43\,\Omega, \tag{19}$$

$$R = 106.7\,\Omega,\ L = 46.5\text{ nH},\ C = 0.22\text{ pF}. \tag{20}$$

Fig. 4 delineates the results of even- and odd-mode analysis applied to dual-band unequal power divider. A RT/Druid 5880 substrate having permittivity 2.2 and thickness 1.5 mm has been employed. In addition, two EM simulation tools such as circuit simulator (ADS) and full-EM simulator (CST MWS) based on FDTD algorithm have been adopted for optimization and systematical design procedure. As the estimated and measured results, it is seen from Fig. 4 that the S21 and S31 of the proposed divider are $-2$ dB and $-5$ dB at both frequencies $f_1 = 1$ GHz and $f_2 = 2.5$ GHz, respectively, with high isolation level characteristics between two output ports. In addition, it is checked out that the phase is equal at ports 2 and 3. There is a little deviation in resonant frequencies between two simulation results of ADS and CST MWS. It is conjectured that the reason is due to the transmission line coupling near the passive components and the discontinuities in T-junction with different line widths in EM simulation and measurements.

## IV. Conclusion

A theoretical investigation for deriving the analytically exact formulas of unequal power division working at dual-band with arbitrary frequency ratios has been presented in this letter. Using even- and odd-mode analysis with isolation resistor, inductor and capacitor, an unequal power division in Wilkinson power divider has been investigated, simulated, and measured with the help of the rigorous analysis and the commercially available full-EM software. In addition, it is guaranteed that the proposed structure and analysis should not be limited to the symmetrical structure and could be extended to the different power splitting ratios and arbitrary frequency ratios in the selected two different frequencies.

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